Lower bounds.

1. Is there a parameterized reduction from Vertex Cover to Independent Set?

2. In the Multicolored Biclique problem the input consists of a bipartite graph G with bipartition classes A, B, an integer k, a partition of A into k sets $A_1, A_2, \ldots, A_k$, and a partition of B into k sets $B_1, B_2, \ldots, B_k$, the question is whether there exists a subgraph of G isomorphic to the biclique $K_{k,k}$, with one vertex in each of the sets A_i and B_i. Prove that Multicolored Biclique is $W[1]$-hard.

3. Given a graph G and an integer k, the Induced Matching problem asks for an induced matching of size k, that is, k edges $x_1y_1, \ldots, x_ky_k$ such that the $2k$ endpoints are all distinct and there is no edge between $\{x_i, y_i\}$ and $\{x_j, y_j\}$ for any $i \neq j$. Prove that Induced Matching is $W[1]$-complete.

4. In the Long Induced Path problem the input consists of a graph G and an integer k, and the question is whether G contains the path on k vertices as an induced subgraph. Prove that this problem is $W[1]$-hard when parameterized by k.

5. Given a graph G and an integer k, the Independent Dominating Set problem asks for a set of exactly k vertices that is both an independent set and a dominating set. Prove that Independent Dominating Set is $W[2]$-complete.

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