

Information Retrieval

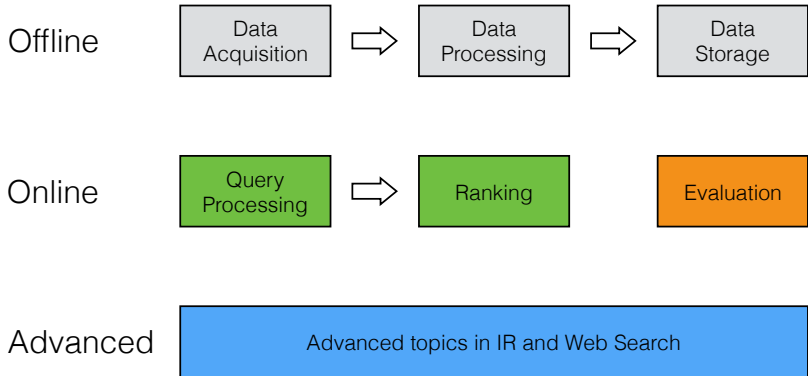
Click Models

Ilya Markov

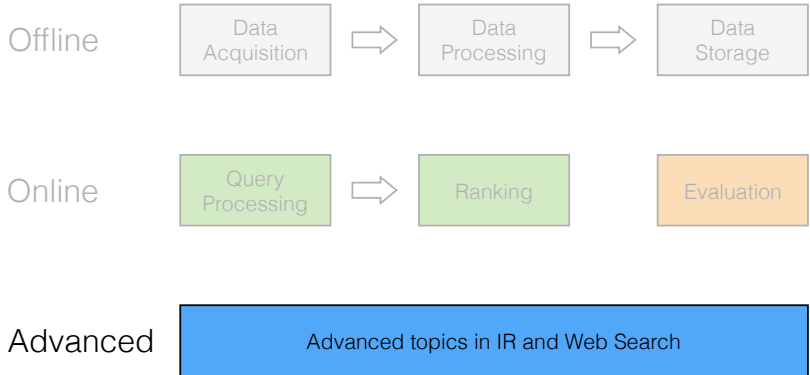
i.markov@uva.nl

University of Amsterdam

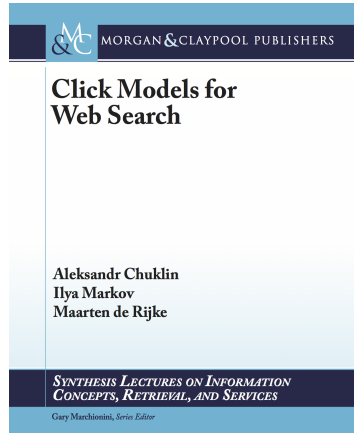
Course overview



Advanced topics in IR



The book



<http://clickmodels.weebly.com/the-book.html>

Tutorials

- SIGIR 2015, Santiago, Chile
- AINL-ISMW FRUCT 2015, St. Petersburg, Russia
- WSDM 2016, San Francisco, USA
- RuSSIR 2016, Saratov, Russia

`http://clickmodels.weebly.com/tutorials.html`

Outline

- 1 Introduction
- 2 Basic click models
- 3 Summary

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Why click models?

Google search results for "wsdm 2016". The search bar contains "wsdm 2016". Below the search bar are tabs for "All", "News", "Images", "Videos", "Shopping", "More", and "Search tools". The search results show "About 61,200 results (0,44 seconds)".

The first result is "WSDM 2016 - San Francisco, USA, Feb 22-25, 2016." with a hand cursor clicking on the link. Below this result are sections for "Attending", "Call for Papers", "Workshops", and "Sponsors".

The second result is "Web Search and Data Mining: The ACM WSDM Conference ..." with a hand cursor clicking on the link.

The third result is "Home - 2016 WSDM Cup Challenge" with a hand cursor clicking on the link.

The fourth result is "Log in to EasyChair for WSDM 2016" with a hand cursor clicking on the link.

At the bottom, there are "Searches related to wsdm 2016" including "sigir 2016", "wsdm 2016 accepted papers", "icde 2016", "wsdm 2014", "wsdm 2017", "what is wsdm", "wsdm 2015", and "wsdm acceptance rate".

At the very bottom is the "Goooooooooogle" logo with a "Next" button.

Why click models?

Scientific modelling is a scientific activity, the aim of which is to make a particular part or feature of the world easier to understand, define, quantify, visualize, or simulate by referencing it to existing and usually commonly accepted knowledge.

Wikipedia, Scientific modelling

Why click models?

Click models make **user clicks** in web search easier to **understand, define, quantify, visualize, or simulate** using (mostly) **probabilistic graphical models**.

Click log

0	0	Q	8	0	7	103	51	92	43	12	73	69	27	105
0	36	Q	174	0	1625	1627	1623	1626	1624	1622	1619	1621	1620	1618
0	50	Q	227	0	2094	2091	2087	2089	2093	2088	2090	2092	2095	2086
0	515	Q	174	0	1625	1627	1623	1626	1624	1622	1619	1621	1620	1618
0	524	Q	1974	0	17562	1627	1626	1623	2091	17559	17563	17558	17561	17560
0	527	C	17562											
0	528	C	1627											
0	529	C	1626											
1	0	Q	9	0	13	70	66	94	50	104	29	21	89	85
1	20	C	104											
1	123	C	21											
1	291	Q	1324	0	11807	11805	11812	11813	11804	11809	11806	11811	11808	11810
1	301	C	11813											
1	8605	C	11808											
1	8737	C	11810											
1	8884	C	11811											
2	0	Q	7	0	77	93	55	86	64	67	76	98	18	54
2	11	C	18											
2	1122	Q	4088	0	35554	35561	35562	35556	35557	35567	35550	35566	35568	35553
2	1127	C	35561											
2	1645	Q	5863	0	36505	36514	36508	36509	50480	36510	36507	50482	50483	50481
2	1646	C	36505											

Yandex Relevance Prediction Challenge
<http://imat-relpred.yandex.ru/en>

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- 1 Introduction
- 2 Basic click models
 - Random click model
 - Position-based model
 - Cascade model
 - Click probabilities
 - Evaluation
 - Parameter estimation
- 3 Summary

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Random click model


Yandex — 62 million answers


Web **San Francisco Travel**
[sanfrancisco.travel](#) 
 San Francisco is home to a bit of everything. Whether you're a first time visitor or a long-time local. This is the place to find out about all things San Francisco.


Images

Video

Translate **San Francisco - Wikipedia, the free encyclopedia**
[en.wikipedia.org](#) > [San Francisco](#) 
 San Francisco (/sæn frənˈsɪskəʊ/), officially the City and County of San Francisco, is the cultural, commercial, and financial center of Northern California and the only consolidated city-county in California.

More **San Francisco travel guide - Wikitravel**
[wikitravel.org](#) > [en/San Francisco](#) 
 San Francisco is a major city in California, the centerpiece of the Bay Area, well-known for its liberal community, hilly terrain, Victorian architecture, scenic beauty, summer fog, and great ethnic and cultural diversity.

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 The job market may seem to navigate these days, but employment and career opportunities can be found in San Francisco's Financial District and Silicon Valley's...

City and County of San Francisco
[sfgov.org](#) 
 SFGov Visitors Key Services: SF Travel Resources. ... Table of links to San Francisco districts and supervisors. District. Supervisor.

P_{click}

P_{click}

P_{click}

P_{click}

P_{click}

P_{click}

Random click model

- Terminology
 - C_u – binary random variable denoting a click on document u
- Random click model (RCM)
 - Any document can be clicked with the same (fixed) probability

$$P(C_u = 1) = \text{const} = \rho$$

Random click model





Search

Web

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$$P(C_{u_1} = 1) = \rho$$

$$P(C_{u_2} = 1) = \rho$$

$$P(C_{u_3} = 1) = \rho$$

$$P(C_{u_4} = 1) = \rho$$

$$P(C_{u_5} = 1) = \rho$$

$$\rho = \frac{\# \text{ clicks}}{\# \text{ shown docs}}$$

CTR models

Random click model (global CTR):

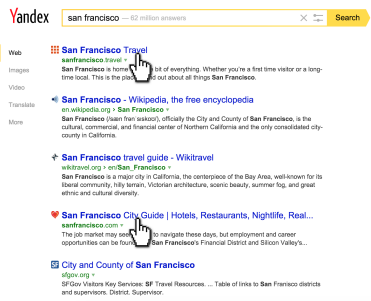
$$P(C_u = 1) = \rho$$

Rank-based CTR:

$$P(C_{u_r} = 1) = \rho_r$$

Query-document CTR:

$$P(C_u = 1) = \rho_{uq}$$



Yandex — 62 million answers

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 sanfrancisco.travel +
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Position-based model

Yandex ✕ ↔ Search


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$$P_{read}(1), P_{click}(u_1q)$$

$$P_{read}(2), P_{click}(u_2q)$$

$$P_{read}(3), P_{click}(u_3q)$$

$$P_{read}(4), P_{click}(u_4q)$$

$$P_{read}(5), P_{click}(u_5q)$$

Position-based model: examination

- Terminology
 - Examination = reading a **snippet**
 - E_r – binary random variable denoting examination of a snippet at rank r
- Position-based model (PBM)
 - Examination depends on rank

$$P(E_r = 1) = \gamma_r$$

Position-based model



Web

San Francisco Travel

sanfrancisco.travel

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$$\gamma_1, P_{click}(u_1q)$$

$$\gamma_2, P_{click}(u_2q)$$

$$\gamma_3, P_{click}(u_3q)$$

$$\gamma_4, P_{click}(u_4q)$$

$$\gamma_5, P_{click}(u_5q)$$

Position-based model: attractiveness

- Terminology
 - Attractiveness = a user wants to click on a document after examining (reading) its snippet
 - A_{uq} – binary random variable showing whether document u is attractive to a user, given query q
- Position-based model (PBM)
 - Attractiveness depends on a query-document pair

$$P(A_{uq} = 1) = \alpha_{uq}$$

Position-based model

Yandex

san francisco — 62 million answers



Search

Web

San Francisco Travel

[sanfrancisco.travel](#) ▾

San Francisco is home to a bit of everything. Whether you're a first time visitor or a long-time local. This is the place to find out about all things **San Francisco**.

$$\gamma_1, \alpha_{u_1 q}$$

Images

Video

Translate

More

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$$\gamma_2, \alpha_{u_2 q}$$

San Francisco travel guide - Wikitravel

[wikitravel.org](#) > [en/San_Francisco](#) ▾

San Francisco is a major city in California, the centerpiece of the Bay Area, well-known for its liberal community, hilly terrain, Victorian architecture, scenic beauty, summer fog, and great ethnic and cultural diversity.

$$\gamma_3, \alpha_{u_3 q}$$

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[sanfrancisco.com](#) ▾

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$$\gamma_4, \alpha_{u_4 q}$$

City and County of San Francisco

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$$\gamma_5, \alpha_{u_5 q}$$

Position-based model: summary

$$P(E_{r_u} = 1) = \gamma_{r_u}$$

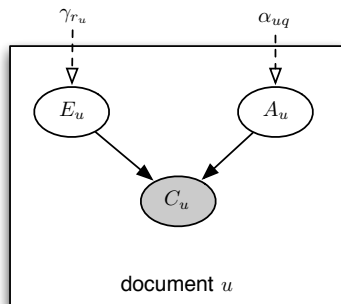
$$P(A_u = 1) = \alpha_{uq}$$

$$P(C_u = 1) = P(E_{r_u} = 1) \cdot P(A_u = 1)$$

Yandex search results for "san francisco" (62 million answers).

- Web**
 - San Francisco Travel**
 - sanfrancisco.travel
 - San Francisco is home to a bit of everything. Whether you're a first time visitor or a long-time local. This is the place to find out about all things San Francisco.
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Position-based model: probabilistic graphical model



Position-based model: exercises

$$P(E_{r_u} = 1) = \gamma_{r_u}$$

$$P(A_u = 1) = \alpha_{uq}$$

$$P(C_u = 1) = P(E_{r_u} = 1) \cdot P(A_u = 1)$$

$$E_{r_u} = 0 \Rightarrow C_u = 0$$

$$A_u = 0 \Rightarrow C_u = 0$$

$$E_{r_u} = 1 \Rightarrow (C_u = 1 \iff A_u = 1)$$

$$A_u = 1 \Rightarrow (C_u = 1 \iff E_{r_u} = 1)$$

Yandex search results for "san francisco" (62 million answers).

- Web:** [San Francisco Travel](#) (sanfrancisco.travel) - San Francisco is home to a bit of everything. Whether you're a first time visitor or a long-time local. This is the place to find out about all things San Francisco.
- Images:** (Thumbnail)
- Video:** (Thumbnail)
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Position-based model

$$P(E_{r_u} = 1) = \gamma_{r_u}$$

$$P(A_u = 1) = \alpha_{uq}$$

$$P(C_u = 1) = P(E_{r_u} = 1) \cdot P(A_u = 1)$$

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Cascade model

- 1 Start from the first document
- 2 Examine documents one by one
- 3 If click, then stop
- 4 Otherwise, continue

The screenshot shows a Yandex search engine interface. The search bar contains the text "san francisco" and indicates "82 million answers". Below the search bar, there are several search results listed under the "Web" category. A mouse cursor is pointing at the first result, "San Francisco Travel", which includes a small "sanfrancisco.travel" logo and a brief description: "San Francisco is home to a bit of everything. Whether you're a first time visitor or a long-time local. This is the place to find out about all things San Francisco." Other visible results include "San Francisco - Wikipedia, the free encyclopedia", "San Francisco travel guide - Wikitravel", "San Francisco City Guide | Hotels, Restaurants, Nightlife, Real...", and "City and County of San Francisco".

Cascade model

$$E_r = 1 \text{ and } A_{u_r} = 1 \Leftrightarrow C_r = 1$$

$$P(A_{u_r} = 1) = \alpha_{u_r} q$$

$$\underbrace{P(E_1 = 1)} = 1$$

start from first

$$\underbrace{P(E_r = 1 \mid E_{r-1} = 0)} = 0$$

examine one by one

$$\underbrace{P(E_r = 1 \mid C_{r-1} = 1)} = 0$$

if click, then stop

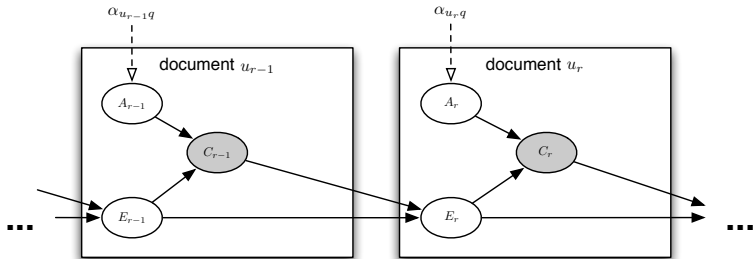
$$\underbrace{P(E_r = 1 \mid E_{r-1} = 1, C_{r-1} = 0)} = 1$$

otherwise, continue

The screenshot shows a Yandex search engine interface. The search bar contains the text "san francisco" and indicates "62 million answers". Below the search bar, there are several search results:

- Web:** "San Francisco Travel" with a sub-link "sanfrancisco.travel". A mouse cursor is hovering over this result.
- Images:** "San Francisco is home to a lot of everything. Whether you're a first time visitor or a long-time local. This is the place to find out about all things San Francisco."
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- More:** "City and County of San Francisco" with a sub-link "sfgov.org >".
- More:** "SF Gov Visitors Key Services, SF Travel Resources. ... Table of links to San Francisco districts and supervisors. District Supervisor."

Cascade model: probabilistic graphical model



Basic click models summary

- CTR models
 - + count clicks (simple and fast)
 - do not distinguish examination and attractiveness
- Position-based model (PBM) → User browsing model
 - + examination and attractiveness
 - examination of a document at rank r does not depend on examinations and clicks above r
- Cascade model (CM) → Dynamic Bayesian network
 - + cascade dependency of examination at r on examinations and clicks above r
 - only one click is allowed

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Click probabilities

- Full probability – probability that a user clicks on a document at rank r

$$P(C_r = 1)$$

- Conditional probability – probability that a user clicks on a document at rank r given previous clicks

$$P(C_r = 1 \mid C_1, \dots, C_{r-1})$$

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Click probabilities

- Full probability

$$P(C_{r+1} = 1) = \alpha_{u_{r+1}q} \epsilon_r \cdot \left(\begin{array}{l} P(E_{r+1} = 1 \mid E_r = 1, C_r = 1) \cdot P(C_r = 1 \mid E_r = 1) \\ + P(E_{r+1} = 1 \mid E_r = 1, C_r = 0) \cdot P(C_r = 0 \mid E_r = 1) \end{array} \right)$$

- Conditional probability

$$P(C_{r+1} = 1 \mid C_1, \dots, C_r) = \alpha_{u_{r+1}q} \cdot \left(\begin{array}{l} P(E_{r+1} = 1 \mid E_r = 1, C_r = 1) \cdot c_r^{(s)} \\ + P(E_{r+1} = 1 \mid E_r = 1, C_r = 0) \cdot \frac{\epsilon_r(1 - \alpha_{u_rq})}{1 - \alpha_{u_rq}\epsilon_r} \cdot (1 - c_r^{(s)}) \end{array} \right)$$

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Evaluation

Click model's output	Evaluation
Full click probabilities	Perplexity
Conditional click probabilities	Log-likelihood

Perplexity

Perplexity measures how well a click model estimates full click probabilities (i.e., when clicks are not observed).

$$p_r(M) = 2^{-\frac{1}{|\mathcal{S}|} \sum_{s \in \mathcal{S}} \overbrace{(\log_2 P_M(C_r^{(s)} = c_r^{(s)}))}^{\text{full click probability}})$$

$$p_r(M) \in [1..2]$$

Likelihood

Likelihood measures how well a click model estimates conditional click probabilities given observed clicks.

$$\begin{aligned}\mathcal{LL}(M) &= \frac{1}{|\mathcal{S}|} \sum_{s \in \mathcal{S}} \log P_M \left(C_1 = c_1^{(s)}, \dots, C_n = c_n^{(s)} \right) \\ &= \frac{1}{|\mathcal{S}|} \sum_{s \in \mathcal{S}} \sum_{r=1}^n \underbrace{\log P_M \left(C_r = c_r^{(s)} \mid \mathbf{C}_{<r} = \mathbf{c}_{<r}^{(s)} \right)}_{\text{conditional click probability}}\end{aligned}$$

$$\mathcal{LL}(M) \in [-\infty..0]$$

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Parameter estimation

- Maximum likelihood estimation
- Expectation-maximization

MLE for random click model

$$P(C_u = 1) = \rho$$

$$\mathcal{L} = \underbrace{\prod_{s \in \mathcal{S}} \prod_{u \in \mathcal{S}} \rho^{c_u^{(s)}} (1 - \rho)^{1 - c_u^{(s)}}}_{\text{likelihood of Bernoulli random variable}}$$

$$\mathcal{LL} = \sum_{s \in \mathcal{S}} \sum_{u \in \mathcal{S}} \left(c_u^{(s)} \log(\rho) + (1 - c_u^{(s)}) \log(1 - \rho) \right)$$

$$\rho = \frac{\sum_{s \in \mathcal{S}} \sum_{u \in \mathcal{S}} c_u^{(s)}}{\sum_{s \in \mathcal{S}} |s|} = \frac{\# \text{ clicks}}{\# \text{ shown docs}}$$

Expectation maximization

- ① Set parameters to some initial values
- ② Repeat until convergence
 - E-step: derive the expectation of the likelihood function
 - M-step: maximize this expectation

Expectation maximization

$$\begin{aligned}
 Q(\theta_c) &= \sum_{s \in \mathcal{S}} \mathbb{E}_{\mathbf{X} | \mathbf{C}^{(s)}, \Psi} \left[\log P(\mathbf{X}, \mathbf{C}^{(s)} | \Psi) \right] \\
 &= \sum_{s \in \mathcal{S}} \mathbb{E}_{\mathbf{X} | \mathbf{C}^{(s)}, \Psi} \left[\sum_{c_j \in \mathcal{S}} \left(\mathcal{I}(X_{c_j}^{(s)} = 1, \mathcal{P}(X_{c_j}^{(s)}) = \mathbf{p}) \log(\theta_c) + \mathcal{I}(X_{c_j}^{(s)} = 0, \mathcal{P}(X_{c_j}^{(s)}) = \mathbf{p}) \log(1 - \theta_c) \right) + \mathcal{Z} \right] \\
 &= \sum_{s \in \mathcal{S}} \sum_{c_j \in \mathcal{S}} \left(P(X_{c_j}^{(s)} = 1, \mathcal{P}(X_{c_j}^{(s)}) = \mathbf{p} | \mathbf{C}^{(s)}, \Psi) \log(\theta_c) + P(X_{c_j}^{(s)} = 0, \mathcal{P}(X_{c_j}^{(s)}) = \mathbf{p} | \mathbf{C}^{(s)}, \Psi) \log(1 - \theta_c) \right) + \mathcal{Z}
 \end{aligned}$$

$$ESS(x) = \sum_{s \in \mathcal{S}} \sum_{c_j \in \mathcal{S}} P(X_{c_j}^{(s)} = x, \mathcal{P}(X_{c_j}^{(s)}) = \mathbf{p} | \mathbf{C}^{(s)}, \Psi)$$

$$\frac{\partial Q(\theta_c)}{\partial \theta_c} = \sum_{s \in \mathcal{S}} \sum_{c_j \in \mathcal{S}} \left(\frac{P(X_{c_j}^{(s)} = 1, \mathcal{P}(X_{c_j}^{(s)}) = \mathbf{p} | \mathbf{C}^{(s)}, \Psi)}{\theta_c} - \frac{P(X_{c_j}^{(s)} = 0, \mathcal{P}(X_{c_j}^{(s)}) = \mathbf{p} | \mathbf{C}^{(s)}, \Psi)}{1 - \theta_c} \right) = 0$$

$$\begin{aligned}
 \theta_c^{(t+1)} &= \frac{\sum_{s \in \mathcal{S}} \sum_{c_j \in \mathcal{S}} P(X_{c_j}^{(s)} = 1, \mathcal{P}(X_{c_j}^{(s)}) = \mathbf{p} | \mathbf{C}^{(s)}, \Psi)}{\sum_{s \in \mathcal{S}} \sum_{c_j \in \mathcal{S}} \sum_{x=0}^1 P(X_{c_j}^{(s)} = x, \mathcal{P}(X_{c_j}^{(s)}) = \mathbf{p} | \mathbf{C}^{(s)}, \Psi)} \\
 &= \frac{\sum_{s \in \mathcal{S}} \sum_{c_j \in \mathcal{S}} P(X_{c_j}^{(s)} = 1, \mathcal{P}(X_{c_j}^{(s)}) = \mathbf{p} | \mathbf{C}^{(s)}, \Psi)}{\sum_{s \in \mathcal{S}} \sum_{c_j \in \mathcal{S}} P(\mathcal{P}(X_{c_j}^{(s)}) = \mathbf{p} | \mathbf{C}^{(s)}, \Psi)} = \frac{ESS^{(t)}(1)}{ESS^{(t)}(1) + ESS^{(t)}(0)}
 \end{aligned}$$

Outline

- 1 Introduction
- 2 Basic click models
- 3 Summary**

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Materials

- Aleksandr Chuklin, Ilya Markov, Maarten and de Rijke
Click Models for Web Search
Morgan & Claypool, 2015

Advanced topics in IR

