~~Пусть целей для~~

$x \in L$ — ~~верный~~ ответ с вер p.

$x \notin L$ — всегда верный ответ

$(1-p)^{\frac{1}{p}}$ раз повторить. и будет вероятность

стоимость $1 - \frac{1}{e}$.

Min CuT

Def Min CuT.

Algo: Take ~~an~~ arbitrarily edge e

Contract edge e

Continue until 2 vertices lef

Number of edges between ~~the~~ vertices

Is an answer.

Analysis If Min CuT $= k \Rightarrow$

$\forall u \ \deg u \geq k \Rightarrow |E| \geq \frac{n \cdot k}{2} \Rightarrow$

$Prob(e \notin \text{Min CuT}) \geq 1 - \frac{k}{\frac{nk}{2}} = 1 - \frac{2}{n} = \frac{n-2}{n}$

$Prob(\mathcal{E}_{n-1}) = Prob(\mathcal{E}_{n-1} | \mathcal{E}_{n-2}) \cdot P(\mathcal{E}_{n-2} | \mathcal{E}_{n-3}) \cdot$

$\ldots P(\mathcal{E}_1 | \mathcal{E}_0) \cdot P(\mathcal{E}_0) =$

$= \frac{n-2}{n} \cdot \frac{n-3}{n-1} \cdot \ldots \cdot \frac{2}{4} \cdot \frac{1}{3} = \frac{2}{n(n-1)}$

Repeat Algo $\frac{n(n-1)}{2}$ times and success is

$1 - \frac{1}{e}$.

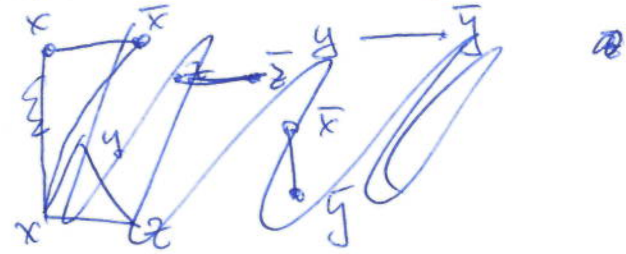
Def Feedback Vertex Set.

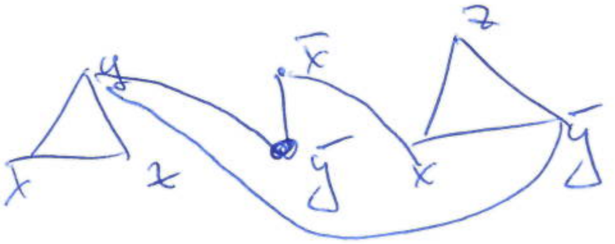
~~Problem Find Feedback~~

3SAT → (Inds Set) → Vertex Cover → Feedback Vertex Set

$(x \vee y \vee z) \wedge (\bar{x} \vee \bar{y})$

$x \vee z \vee \bar{y}$



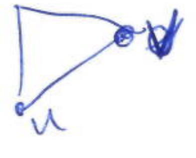


↓

$VC = G \setminus I$

↓

for each edge $e = uv$ create 3 vertices e, u, v



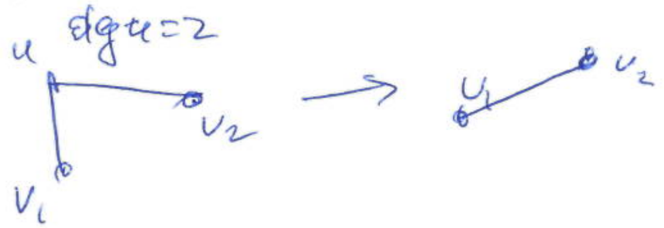
To destroy all cycles we need to destory all Δ. Stupid to take e instead of u or v. vertex cover in original graph correspond to FVS in new graph.

FVS is NP-complet. Hence no poly algo most probably.

We build 4^k algo to look for a FVS of size k.

<u>Reduction Rule 1</u> Delete Висячую вершину

<u>Reduction Rule 2</u>



$\forall u \in G \ \deg u \geq 3$.

Forest $= V_1 \cup V_2 \cup V_{\geq 3}$

$E(Sol, Forest) \geq 2|V_1| + |V_2|$

$E(Forest) \leq |V_1| + |V_2| + |V_{\geq 3}|$

$\vee\!/$

$\dfrac{|V_1| + 2|V_2| + 3|V_{\geq 3}|}{2}$ $\Rightarrow |V_{\geq 3}| \leq |V_1|$

$\Rightarrow E(Sol, Forest) \geq E(Forest)$

Take Random edge e.

Take Random end of e and put it into solution

Continue.

Success with probability $\frac{1}{4^k}$.

1) QuickSort — Las Vegas
2) MinCut — Monte Carlo.
3) FVS

RP (Randomized Polynomial time)

L s.t.: M runs in poly time.

$x \in L \Rightarrow Pr(M(x) \text{ accepts}) \geq \frac{1}{2}$

$x \notin L \Rightarrow Pr(M(x) \text{ accepts}) = 0.$

Probabilistic Turing Machine

Two transition functions δ_1, δ_0.

Choosing with prob $\frac{1}{2}$.

Runs in $T(|x|)$ time if for $\forall$ random choice stops in $T(|x|)$ steps.

Zero-sided error

Las Vegas with poly time

$ZPP = \bigcup_{c>0} ZTime(n^c)$

Expectation of working time is

RTime

$> \frac{1}{2}$

$< \frac{1}{2}$

PP

BPP. $x \in L$ $Pr(\text{accepts}) \geq \frac{3}{4}$

$x \notin L$ $Pr(M(x) \text{ accepts}) \leq \frac{1}{4}$

Primality Testing is easy in random
"Hard" in non-random way. still poly time.

Do we need Randomness ? Yes

1) Seems yes because of ↗ (poly time)

2) Uniformity is important for this matter.

Def Circuit → Acyclic graph, (∧, ∨, ¬) → gates.

Def Random Circuit

Random Circuit = Circuit.

Thm: Adleman ② $\Rightarrow RP \subseteq P/poly$ ($\subseteq BPP_{poly} \subseteq$) if time permitted!

① $ZPP = RP \cap coRP$ If time permitted.

$ZPP \subseteq RP \cap coRP$. easy

$ZPP \supseteq RP \cap coRP$.

A_1 A_2