Important cuts II(HW).

1. Reduce Edge Multiway Cut with $T = 3$ to Directed Edge Multiway Cut with $T = 2$.

2. Let G be an undirected graph and let $A, B \subseteq V(G)$ be two disjoint sets of vertices. Let $\Delta(R^{AB}_{min})$ and $\Delta(R^{AB}_{max})$ be the minimum (A, B)-cuts, and let $\Delta(R^{BA}_{min})$ and $\Delta(R^{BA}_{max})$ be the minimum (B, A)-cuts(reversing the role of A and B). Show that $R^{AB}_{min} = V(G) \setminus R^{BA}_{max}$ and $R^{AB}_{max} = V(G) \setminus R^{BA}_{min}$.

3. Recall algorithm running in time $O^*(4^k)$ that finds all important cuts and show that it can actually output a non-important cut.

4. In the Digraph Pair Cut, the input consists of a directed graph G, a designated vertex $s \in V(G)$, a family of pairs of vertices $\mathcal{F} \subseteq \binom{V(G)}{2}$, and an integer k; the goal is to find a set X of at most k edges of G, such that for each pair $\{u, v\} \in \mathcal{F}$, either u or v is not reachable from s in the graph $G - X$. Show an algorithm solving Digraph Pair Cut in time $2^k n^{O(1)}$ for an n-vertex graph G.

5. Consider the following vertex-deletion variant of Digraph Pair Cut with a sink: given a directed graph G, designated vertices $s, t \in V(G)$, a family of pairs of vertices $\mathcal{F} \subseteq \binom{V(G)}{2}$, and an integer k, check if there exists an (s, t)-separator $X \subseteq V(G) \setminus \{s, t\}$ of size at most k such that every pair in $\mathcal{F}$ either contains a vertex of X or contains a vertex that is unreachable from s in $G \setminus X$.

   Show how to solve this vertex-deletion variant in time $2^k n^{O(1)}$ for an n-vertex graph G, using the algorithm of the previous exercise.