Kernelization(HW)

1. In the Set Splitting problem, we are given a family of sets F over a universe U and a positive integer k, and the goal is to test whether there exists a coloring of U with two colors such that at least k sets in F are non monochromatic (that is, they contain vertices of both colors). Show that the problem admits a kernel with at most $2k$ sets and $O(k^2)$ universe size.

2. In the Minimum Maximal Matching problem, we are given an undirected graph G and a positive integer k, and the objective is to decide whether there exists a maximal matching in G on at most k edges. Obtain a polynomial kernel for the problem (parameterized by k).

3. In the Min-Ones-2-SAT problem, we are given a 2-CNF formula ϕ and an integer k, and the objective is to decide whether there exists a satisfying assignment for ϕ with at most k variables set to true. Show that Min-Ones-2-SAT admits a polynomial kernel.

4. In the d-Bounded-Degree Deletion problem, we are given an undirected graph G and a positive integer k, and the task is to find at most k vertices whose removal decreases the maximum vertex degree of the graph to at most d. Obtain a kernel of size polynomial in k and d for the problem. (Observe that Vertex Cover is the case of $d = 0$.)

5. Show a kernel with $O(k^2)$ vertices for the following problem: given a graph G and an integer k, check if G contains a subgraph with exactly k edges, whose vertices are all of odd degree in the subgraph.

6. A set of vertices D in an undirected graph G is called a dominating set if $N[D] = V(G)$. In the Dominating Set problem, we are given an undirected graph G and a positive integer k, and the objective is to test whether there exists a dominating set of size at most k. Show that Dominating Set admits a polynomial kernel on graphs where the length of the shortest cycle is at least 5. (What would you do with vertices with degree more than k? Note that unlike for the Vertex Cover problem, you cannot delete a vertex once you pick it in the solution.)

7. Show that Feedback Vertex Set admits a kernel with $O(k)$ vertices on undirected regular graphs.