

## Разное.

1. Given a Set Cover instance  $(U, F, k)$ , the minimum possible size of a subfamily  $F' \subset F$  that covers  $U$  can be found in time  $2^{|U|}(|U|+|F|)^{O(1)}$ . Prove above mentioned fact.
2. Given a directed graph  $G$ , a set of terminals  $K \subset V(G)$  and a root  $r \in V(G)$ , Directed Steiner Tree asks for a directed tree rooted at  $r$  such that every terminal in  $K$  is reachable from  $r$  on the tree. Obtain a  $3^{Kn^{O(1)}}$  time algorithm for Directed Steiner Tree.
3. In the Max Leaf Subtree problem, we are given a graph  $G$  together with an integer  $k$  and the question is whether there is a subtree  $T$  of  $G$  with at least  $k$  leaves.
  - (a) Show that  $(G, k)$  is a yes-instance if and only if  $K_{1,k}$  (a graph with a center vertex connected to  $k$  degree-one vertices) is a minor of  $G$ .
  - (b) Deduce that Max Leaf Subtree is non-uniformly fixed-parameter tractable, when parameterized by  $k$ .
4. In the Planar Diameter Improvement problem, the input consists of a planar graph  $G$  and an integer  $k$ , and the task is to check if there exists a supergraph of  $G$  that is still planar, and at the same time has diameter at most  $k$ .
  - (a) Prove that this problem is non-uniformly fixed-parameter tractable, when parameterized by  $k$ .
  - (b) Show that it only suffices to consider supergraphs  $G'$  of  $G$  with  $V(G') = V(G)$  that is, it makes sense only to add edges to  $G$ , but adding new vertices does not help.
5. In the Cycle Packing problem the input is a graph  $G$  with an integer  $k$  and the task is to determine whether there exist  $k$  cycles in  $G$  that are pairwise vertex disjoint. Prove that this problem is non-uniformly fixed-parameter tractable, when parameterized by  $k$ .