Kernelization

1. In the Point Line Cover problem, we are given a set of n points in the plane and an integer k, and the goal is to check if there exists a set of k lines on the plane that contain all the input points. Show a kernel for this problem with $O(k^2)$ points.

2. A graph G is a cluster graph if every connected component of G is a clique. In the Cluster Editing problem, we are given as input a graph G and an integer k, and the objective is to check whether one can edit (add or delete) at most k edges in G to obtain a cluster graph. That is, we look for a set $F \subset \binom{V(G)}{2}$ of size at most k, such that the graph $(V(G), E(G)\Delta F)$ is a cluster graph.

   (a) Show that a graph G is a cluster graph if and only if it does not have an induced path on 3 vertices (sequence of three vertices u, v, w such that uv and vw are edges and $uw \notin E(G)$).

   (b) Show a kernel for Cluster Editing with $O(k^2)$ vertices.

3. In the Set Splitting problem, we are given a family of sets F over a universe U and a positive integer k, and the goal is to test whether there exists a coloring of U with two colors such that at least k sets in F are non monochromatic (that is, they contain vertices of both colors). Show that the problem admits a kernel with at most $2k$ sets and $O(k^2)$ universe size.

4. In the Minimum Maximal Matching problem, we are given an undirected graph G and a positive integer k, and the objective is to decide whether there exists a maximal matching in G on at most k edges. Obtain a polynomial kernel for the problem (parameterized by k).

5. In the Min-Ones-2-SAT problem, we are given a 2-CNF formula ϕ and an integer k, and the objective is to decide whether there exists a satisfying assignment for ϕ with at most k variables set to true. Show that Min-Ones-2-SAT admits a polynomial kernel.